EFFECTS OF VIBRATIONS ON MASS TRANSFER FROM A SPHERE AT HIGH PRANDTL NUMBERS

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ABSTRACT: Sonic vibrations are used to intensify diffusion in chemical technology [1]. We have previously [2-4] examined the effects of such waves on transport in gaseous media (Prandtl's number $P \leq 1$). Here we extend the results to heteroganeous mass transfer in liquids (P large).

Mass transfer in this case occurs via internal secondary flows, not external ones. The main calculated results have been tested by experiment.

NOTATION

Here u and v are the longitudinal and transverse components of the velocity; x and y are the longitudinal and transverse coordinates; R is the sphere radius; r is the instantaneous sphere radius; λ is the wavelength of waves; ω is the frequency; B is the amplitude of vibration velocity; s is the amplitude of displacement in waves; m is the dimensionless concentration of diffusing substance; D is the diffusion coefficient; μ is the dynamic viscosity; ρ is the density; ν is the kinematic viscosity; q is the flux of material from surface of sphere; β is the mass-transfer coefficient; N is the Nusselt number; P is the Prandtl number; Γ is the gamma function.

Consider a sphere in a liquid vibrating harmonically. There is mass transfer between the sphere and the liquid, and it is assumed that $P \to \infty$. The assumptions previously made [2-4] still apply:

$$\lambda / R \gg 1, \quad \nu = \text{const}, \quad \mu = \text{const},$$
$$D = \text{const}, \quad \rho = \text{const}.$$

Consider the case where $s/R \ll 1$. Secondary flows then occur around the sphere (Fig. 1), and an analytic expression has been given [3] for these.

The velocity distribution at the surface of the sphere may be put in the form $u = \tau y/\mu$ for $P \to \infty$, in which τ is the local friction, given by expression (2.6) of [3]. This differs from Schlichting's formula in that it contains terms that allow for axial symmetry.

The equation of continuity gives

$$v = -\frac{y^2}{2\mu}\frac{\partial \tau}{\partial x} - \frac{\tau}{2\mu}\frac{\partial \ln r}{\partial x} y^2.$$

We give the following form [2-4] for the diffusion equation and boundary conditions in the xy coordinate system (neglecting the effects of the velocity pulsations on the mean concentration profile):

$$\frac{\partial^2 m}{\partial y^2} + A(x) y^2 \frac{\partial m}{\partial y} = B(x) y \frac{\partial m}{\partial x},$$
$$m = m_1 \quad \text{for } y = 0, \quad m = 0 \quad \text{for } y = \infty,$$
$$A(x) = \frac{1}{2\mu D}\frac{\partial \tau}{\partial x} + \frac{\tau}{2\mu D}\frac{\partial \ln r}{\partial x}, \quad B(x) = \frac{\tau}{\mu D}. \tag{1}$$

Introducing the new independent variable [5]

$$\eta = y\left[e^{-F(x)}\int_0^x \frac{e^{F(x)}}{B(x)}dx\right]^{-1/3}, \qquad F(x) = 3\int_0^x \frac{A(x)}{B(x)}dx, \tag{2}$$

(1) gives us the ordinary differential equation

$$\frac{d^2 m}{d\eta^2} + \frac{\eta^2}{3}\frac{dm}{d\eta} = 0 \qquad \begin{array}{l} m = m_1 \quad \text{for } \eta = 0 \\ m = 0 \quad \text{for } \eta = \infty. \end{array} \tag{3}$$

The integral of (3) that satisfies the boundary conditions is known:

$$m = m_1\left(1 - \frac{\Phi(\eta)}{\Phi(\infty)}\right), \qquad \Phi(\eta) = \int_0^\eta \exp\left(\frac{-\eta^3}{9}\right)d\eta,$$
$$\Phi(\infty) = 3^{-1/3}\Gamma(1/3) = 1.86. \tag{4}$$

For a sphere [3]

$$\tau = \frac{9}{16\sqrt{2}}\frac{B^2\mu}{\sqrt{\omega\nu}R}\sin\frac{2x}{R},$$

so

$$\eta = 0.928\left(\frac{B^2}{\sqrt{\omega\nu}DR^2}\right)^{1/3}\frac{\sin\varphi\cdot\cos^{1/2}\varphi\cdot y}{[1/2(\varphi - \sin\varphi\cos\varphi)]^{1/3}}, \quad \varphi = \frac{x}{R}.$$

We have from (4) that

$$q = -\rho D\left[\left(\frac{\partial m}{\partial \eta}\right)\frac{\partial \eta}{\partial y}\right]_{y=0} =$$
$$= 0.63\,\rho m_1\left(\frac{B^2 D^2}{\sqrt{\omega\nu}R^2}\right)^{1/3}\frac{\sin\varphi\cos^{1/2}\varphi}{[\varphi - \sin\varphi\cos\varphi]^{1/3}}.$$

For N reckoned along the radius we have

$$N = 0.63\,K\frac{\sin\varphi\cos^{1/2}\varphi}{(\varphi - \sin\varphi\cos\varphi)^{1/3}}, \quad K = \left(\frac{B^2 R}{\sqrt{\omega\nu}D}\right)^{1/3}. \tag{5}$$

The average over the sphere for N along a diameter is

$$N_d = \frac{\beta d}{D} = 0.99K. \tag{6}$$

The results were tested via experiments on mass transfer in a benzoic acid-water system. Glass balls coated with a thin layer of benzoic acid were oscillated in water with various frequencies (10-125 Hz) Hz) and amplitudes provided by a crank mechanism. The oscillation amplitude was determined with a KM-6 cathetometer, while the frequency was measured with a chopper disk, photodiode, and ICh-6 frequency meter.

The benzoic acid was coated on the glass balls by dipping into the melted acid, which gave a layer 0.3-0.5 mm thick and of weight about 150 mg. The weight of the specimen with mount was about 1.3 g, while the change in weight during the experiment was 5-10 mg, i.e., within the limits of the balance scale. The spheres were examined under the microscope to select those with the smoothest coating.

The dissolution surface was taken as being the external geometrical surface as deduced from the mean diameter derived from several measurements with a horizontal comparator.

The D for benzoic acid in water was calculated from the formulas of [6].

The local transport coefficients were determined with an IZA-2 horizontal comparator by measuring the diameter before and after the experiment in seven sections. Figure 1 shows these results, in which 1 is the region of internal flow, 2 is the region of external flow, 3 is the loss curve on a scale in which the loss at the flank is taken as 1, and 4 is the direction of oscillation.

The loss differs from that in a gas in being maximal at points lateral to the oscillatory motion, because internal flows predominate, not external ones as for $P \leq 1$. The results agree with calculation from (5).

The total mass-transfer coefficients were determined by weighing; the calculated result from (6) (full line) and the experiments results are shown in Fig. 2.

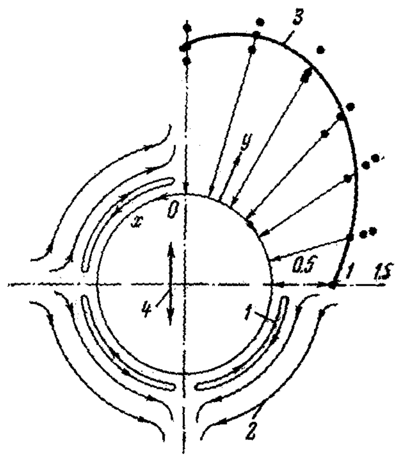

Fig. 1

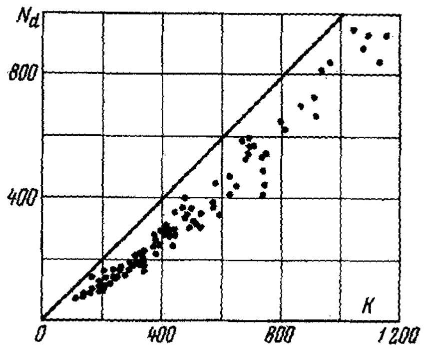

Fig. 2

The results imply that the convective diffusion for P large and $s/R \ll \ll 1$ is determined by the internal region of secondary flows.

Relation (5) differs substantially from analogous relations derived previously [2–4] for heat transfer for $P \leq 1$.

The results may be used to evaluate the effects of vibration on transport in liquids.

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